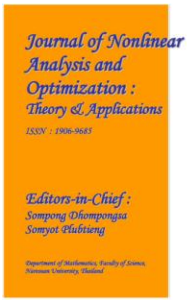

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DESIGNING AND ANALYSING SPUR GEARS WHICH ARE SYMMETRICAL AND ASYMMETRIC IN SHAPE

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ABSTRACT—
Without gears, power cannot be transferred from one area of a machine to another. It is feasible to alter the characteristics of power transmission, including input direction, torque, and velocity. Throughout the power transmission process, they are subjected to a range of loads. The gears are severely strained by these stresses. If the strains are higher than what the gear's surface can sustain, failures happen. To reduce these stresses, several modifications are made to the gear design. The utilisation of unequal pressure angles between the driving and coast sides is one of these design modifications. This kind of gear is known as a "asymmetric spur gear". Compared to normal spur gears, these asymmetric spur gears can bear less force because to their wide roots teeth.
In this work, we examine spur gear designs that use various module numbers and pressure angles, and we assess and verify spur gear designs using Hertz theory.
This work also includes the design and analysis of asymmetric spur gears with different module combinations and their analytical representation. Ultimately, a comparative analysis of every gear is conducted, followed by the creation of graphs and recommendations.
Pressure angles, tensions in asymmetric spur gears, Hertz theory, and modules are explored.

INTRODUCTION

Gears are the most important and common instruments in present mechanical world for transmitting power. They vary in many sizes starting from smallest gears used in watches to large and huge gears used in heavy machines. They are very vital in any mechanical machines. These are used mainly for varying speeds, power, and also direction of input and output. For the different kinds of use there are different kinds of gears i.e., Bevel gears, helical gears, spur gears, worm gears. Among all these gears spur gears are simple gears. Their design is very simple compared to other gears. These gears while they are operating, they are subjected to different kinds of loads, thus resulting in lots of stresses in gears.
These stresses are of two types bending stresses and contact stresses. Bending stresses are calculated theoretically by using Lewis theory and contact stresses are calculated theoretically by Hertz theory. The gears are defined by many factors like module, pressure angle, pitch circle and many more factors. This paper mainly deals with module and pressure angle. Gears with different modules and pressure angles are designed and analyzed. Conventional gears have similar design on both sides of gears i.e., drive and coast side. They are subjected to many stresses. To reduce these stresses to some extent, we need to alter the design. The design alteration includes different pressure angles on drive and coast side. They are called asymmetric spur gears. Coming to point of discovery of gears it is dated at the time of 4th century BC in China which has been preserved at the Luoyang Museum of Henan Province, China. The earliest preserved gears in Europe were found in the Antikythera mechanism, it is an example of very early and complex design of a gear to calculate the astronomical positions. The time of construction Antikythera mechanism is now estimated between 150 and 100 BC. These gears were greatly developed by the then Greek polymath Archimedes (287–212 BC).

I. DESIGN OF SYMMETRIC SPUR GEARS

Design of symmetric spur gears is done in Catia V5 software. The dimensions required for the design of spur gears are obtained from the formulae [1]. This paper depicts the affects of module and pressure angle on the spur gear. Different modules are used to design a spur gear. Each module is again designed by two kinds of pressure angles i.e., 14.5 and 20. Modules are selected through the table given below [1]. Four modules are taken from choice1 and one module is taken from choice-2.

TABLE I

DIFFERENT KINDS OF MODULES

Choice	1.0	1.25	1.5	2.0	2.5	3.0	4.0
1	5.0	6.0	8.0	10	12	16	20
Choice	1.125	1.375	1.75	2.25	2.75	3.5	4.5
2	5.5	7	9	11	14	18	

The modules used in this paper are highlighted i.e., 2.5, 3.5, 3.0, 4.0, 5.0.

Calculations:

a. Number of teeth on pinion = Np = 50

b. Number of teeth on gear = Ng = 50

c. Pressure angle = α = 20

d. Module = m = 3.5mm

e. Pitch circle diameter = Dp = m×N = 175mm

f. Base circle diameter = Db = Dp×cos(α)=164.45mm

g. Addendum circle diameter = Da= Dp+2m= 182mm

h. Dedendum circle diameter = Dd= Dp- (2+π/N)×m = 167.78mm

i. Face width = b= 10×m = 35mm

j. Fillet radius = rp= 0.4×m = 1.4mm

These are the formulae for obtaining dimensional values of a symmetric spur gear with module 3.5 and 20°pressure angle. In the similar way, different modules and pressure angles are taken and calculated to obtain the dimensional values. These values are used for the design of different symmetric spur gears in Catia V5.

The validation of the spur gears is done using Hertz theory. Those calculations are depicted below [1], [2].

Earle Buckingham used the hertz theory to determine the contact stress between a pair of teeth while transmitting power by treating the pair of teeth in contact as cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point. According to hertz theory [1], when two cylinders are in contact with each other, the contact stress is given by,

$$\sigma c = 2P/\pi BL \quad \text{(i)}$$

$$\text{And } B = \left[\frac{2P\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)}{\pi L\left(\frac{1}{d_1} + \frac{1}{d_2}\right)} \right] \quad \text{(ii)}$$

σc = Maximum contact stress (N/mm2)

P = Force acting between both the cylinders (N) B = Half width of deformation (mm)

L = Axial length of cylinders (mm)

d1, d2 = diameters of two cylinders (mm) E1, E2 = modulus of elasticity of two cylinder materials (N/mm2) μ1, μ2 = Poisson’s ratio of two cylinder materials.

TABLE II
TOLERANCES ON THE ADJACENT PITCH

Grade	*e*(microns)
1	0.80+0.06Ø
2	1.25+0.10Ø
3	2.00+0.16Ø
4	3.20+0.25Ø
5	5.00+0.40Ø
6	8.00+0.63Ø
7	11.00+0.90Ø
8	16.00+1.25Ø
9	22.00+1.80Ø
10	32.00+2.50Ø
11	45.00+3.55Ø
12	63.00+5.00Ø

From [1] the error e is given by,
Where,
m = module of gear (mm)
dp = pitch circle diameter gear (mm)
In our case we have,
m = 3.5mm and dp = 175mm Therefore we have,
φ= 3.5+0.25 175 (xvi)
e = ep + eg (xvii)
Where, ep = error for pinion , eg = error for gear,
Since, in our case the pinion and gear are of equal geometry, therefore tolerance factor φ is same for both gear and pinion. Also the grades listed in table 2 from grade 1 to grade 12 are arranged in decreasing order of precision. Considering the gear and pinion to be of grade 1 which is of top precision, we have ep = eg = 0.80 + 0.06 (xviii)
From equations (xvii) & (xviii) we have, e = 2ep = 2eg = 2(0.80 + 0.06φ) (xix) From equation (xvi) we have
Substituting this value of in equation (xix) we have, e = 2[0.80 + 0.06( )]
⇒e = 2.417μm = 2.417
Now from equation 3.15 we have, the dynamic load,

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{Ceb + P_t}}$$

Substituting the values of e, Pt, C, v and b in the above the equation we have,
Pd

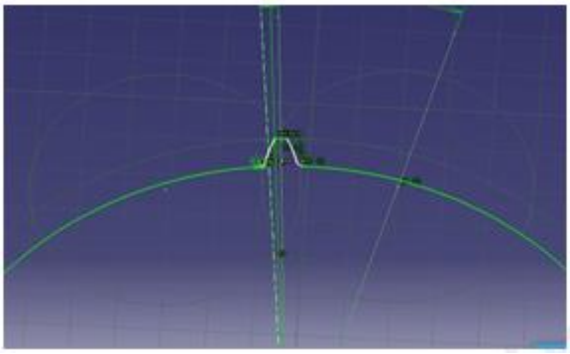
$$\frac{21\times 54.978\times[(10711.5\times 2.417\times 10^{-3}\times 35)+508.571]}{21\times 54.978+\sqrt{(10711.5\times 2.417\times 10^{-3}\times 35)+508.571}}$$

$$\Rightarrow P_d = \frac{21\times 54.978\times 1414.710}{21\times 54.978+\sqrt{1414.710}}$$

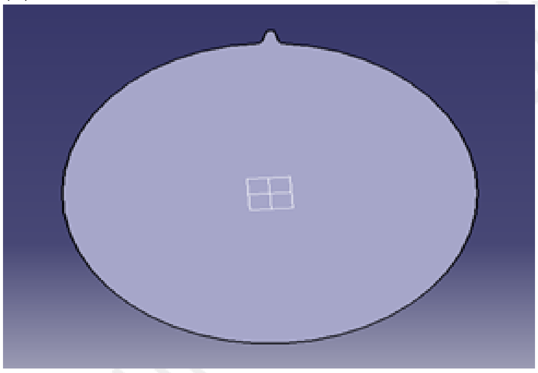
$$\Rightarrow P_d = 1370.0756\ N \qquad \text{(xx)}$$

From equation (xiv) and (xx) we have,
Pts > Pd

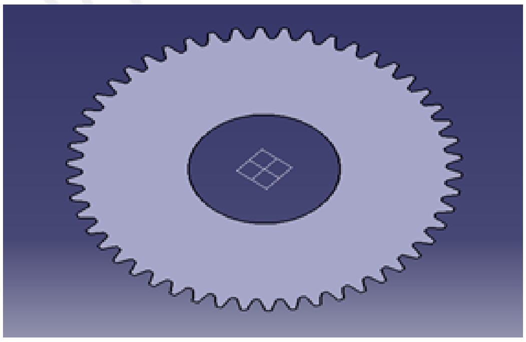
Therefore the design is safe from surface durability considerations. In the same above procedure, calculations for other modules and pressure angles are carried out. The results satisfy the above conditions. The final design and assembly of gear is shown in below figures.



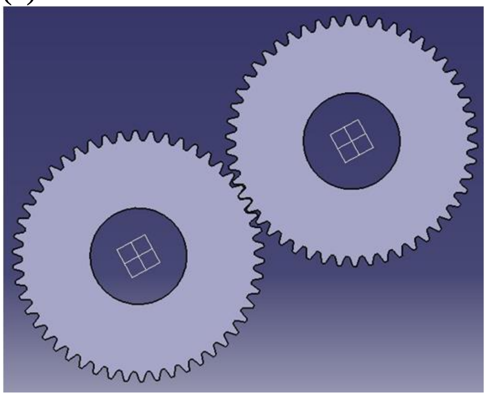
(a)



(b)



(c)



(d)

Fig. 1 (a)showing tooth profile in catia; (b)tooth profile after pad option; (c)tooth profile with 50 teeth; (d)assembly of spur gears

II. DESIGN OF ASYMMETRIC SPUR GEARS

The design of asymmetric spur gears [7] is slightly different from symmetric spur gears. This is due to the reason that these gears have different pressure angles of the drive and coast side.

Due to different pressure angles, these gears have different base circle diameters. These base circle diameters form the main aspect for the design of asymmetric spur gears. Though the base circles are different the centrodes (pitch circles) of both side profiles are similar when meshing of gear and pinion is considered. The centrodes are in tangency to point P which is the instantaneous center of rotation. The line of action is the common tangent to pair of base circles. The pressure angle is the angle between the line of action and the line which is tangent to both the centrodes.

The relation between pinion-gear tooth thicknesses: In case of standard gear drive both the thicknesses of pinion and gear teeth are similar i.e., sp and sg respectively. In these asymmetric spur gears we have similar pinion and gear teeth, so we can take sp and sg as equal. We have a relation between sp and sg . It is represented as

$$\lambda_t = \frac{s_p}{s_g} \qquad \text{(i)}$$

Whereas, ' λ_t is ratio of tooth thickness of pinion to the gear measured on the meshing centrodes of the gears. In this case the asymmetric gears have similar s_p and s_g.

Due to rolling of centrodes, we have

$$sg = wp \qquad \text{(ii)}$$

$$sp = wg \qquad \text{(iii)}$$

Where w_p and w_g are width of space of pinion andgear measured on centrodes.

It is easy to verify that

$$s_p + w_p = s_g + w_g = p_c = \frac{\pi}{P_d} \qquad \text{(iv)}$$

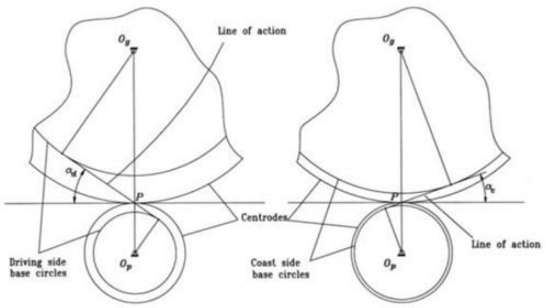


Fig. 2 Representation of line of action, centrodes and base circles in an asymmetric gear.

Where, pc and Pd are circular and diametral pitches respectively.

Then we have the following expressions for sp and s

$$s_p = \frac{\lambda_t \times p_c}{1+\lambda_t} \quad \frac{\lambda_t \pi}{(1+\lambda_t)P_d} \quad t \qquad \text{(v)}$$

$$s_g = \frac{s_p}{\lambda_t} = \frac{p_c}{1+\lambda_t} = \frac{\pi}{(1+\lambda_t)P_d} \qquad \text{(vi)}$$

Since we have λ_t =1; P_d = 1/m; p_c = π x m; in this case, we obtain

$$s_p = s_g = \frac{\pi}{2 \times P_d} = \frac{\pi \times m}{2} = \frac{\pi \times 3.5}{2} = 5.498$$

Radii of base circles: The radius of the base circle and the radius of the pitch circle are related by below equation.

$$r_{bd} = r_p \cos\alpha_d = \frac{N}{2P} \cos\alpha_d \qquad \text{(vii)}$$

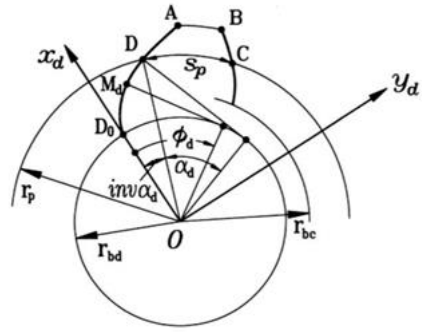
$$r_{bc} = r_p \cos\alpha_c = \frac{N}{2P} \cos\alpha_c \qquad \text{(viii)}$$

From above formulae we have rbd and rbcrbd as 75.777 and 82.223mm respectively.

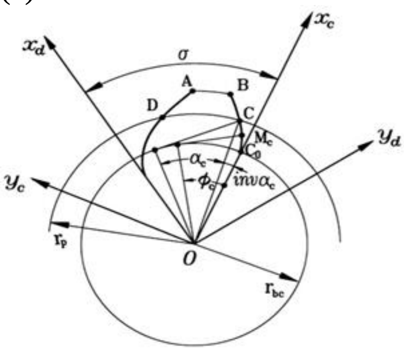
A. Analytical presentation of involute profiles

The figure 3 represents the involute profiles of an asymmetric tooth. The drawings can be either referred to pinion or gear.

We require the following as inputs: (a) DC = sp, the tooth thickness on the pitch circle, (b) radius rp of the pitch circle, (c) pressure angles αd and αc of drive and coast side profiles, (d) radii of base circles rbd and rbc for the drive and coast side profiles. Analytical representation of the asymmetric profile is to be obtained by these values.



(a)



(b)

Fig.3 (a) Representation of drive profile;
(b) Representation of coast profile.

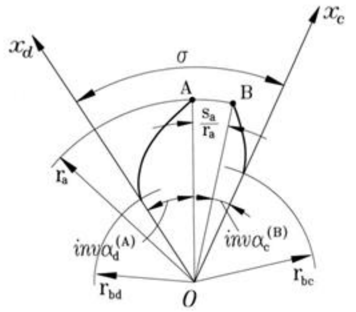


Fig.4 Determination of addendum tooth tip thickness

Step 4: Drawings of fig. yield

$$S_a = r_a\left(\sigma - \mathrm{Inv}\left(\alpha_d^{(A)}\right) - \mathrm{Inv}\left(\alpha_c^{(B)}\right)\right)$$

S_a = 91(0.13149 - 0.078136 - 0.03140) = 1.998

Here: parameter σ can be determined by using equation (xiii), $\mathrm{Inv}\left(\alpha_d^{(A)}\right)$ by using equations (xv) and (xvi), and $\mathrm{Inv}\left(\alpha_c^{(B)}\right)$ by using equation (xvii).

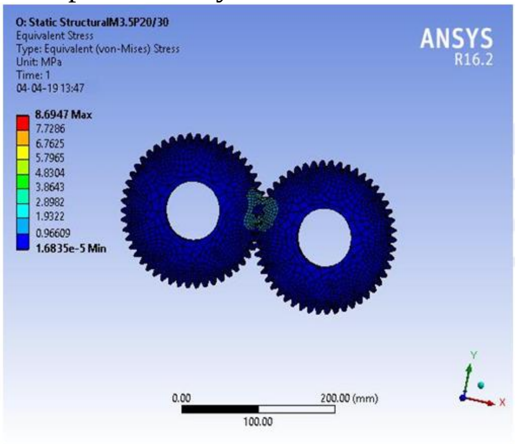
Step 5:

The derived equations enable to obtain radius ra for a pointing tooth taking in equation (xvii) that sa = 0.

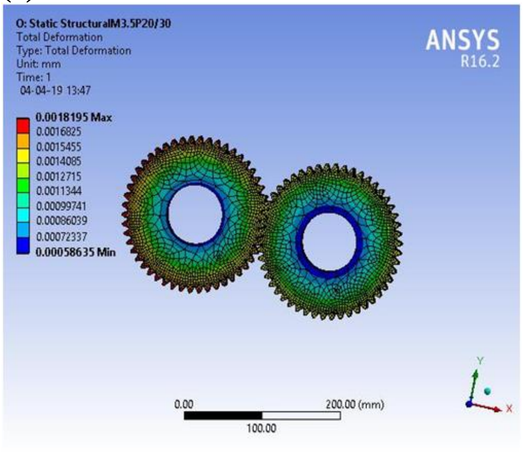
In this similar way the asymmetric gears with different modules are calculated and designed using Catia V5.

III. FINNITE ELEMENT ANALYSIS

The finite element analysis is carried out in ANSYS work bench 16.2. The static analysis is carried out in workbench. The input given is moment for the pinion , which is got from the hertz theory calculations. The moment value is 44500 N-mm. This value is taken for every case in this paper .The gear material used in this analysis is stainless steel. The figures depicted below are some of the examples of analysis done.



(a)



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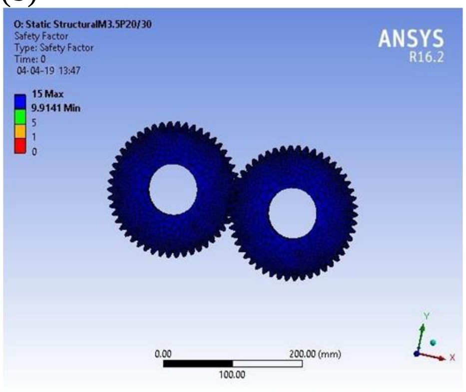


(c)

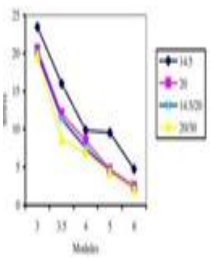
Fig.5 (a) equivalent stress ; (b) Total deformation; (c) safety factor of spur gear with module 3.5 and pressure angle 20

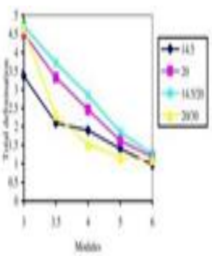
V. RESULTS AND COMPARISONS

The design of the symmetric and asymmetric spur gears is done in Catia, and their analysis is done in ANSYS softwares. The design and analysis are done for five module samples and each module gear is designed with three different pressure angles. Thus we obtain 15 results. The final results in the form of table are as follows.

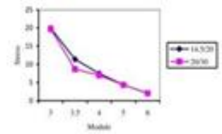
TABLE 2 RESULTS OF ANSYS

Module (mm)	Pressure angle(°)	Stress (Von-mises) (MPa)	Total Deformation (10^3xmm)	Safety factor
3	14.5	23.451	3.3645	3.6757
	20	20.492	4.511	4.2065
	14.5/20	19.933	4.715	4.3244
	20/30	19.695	4.64	4.3767
3.5	14.5	16.017	2.006	5.3816
	20	12.023	3.323	7.1669
	14.5/20	11.372	3.72	7.5801
	23/30	8.6947	2.2998	9.9141
4	14.5	9.8434	1.8984	8.7572
	20	8.4392	2.4586	9.9141
	14.5/20	7.5026	2.8498	11.489
	20/30	6.9936	1.523	12.325
5	14.5	9.5531	1.4054	9.023
	20	4.635	1.6271	15
	14.5/20	4.3911	1.848	15
	20/30	4.3434	1.1763	15
6	14.5	4.7405	0.9796	15
	20	2.464	1.1842	15
	14.5/20	2.0618	1.262	15
	20/30	2.0411	1.098	15

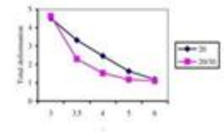




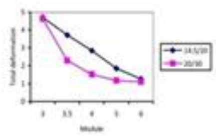
Graph 1 Module v/s Stress with different pressure angled gears Graph 2 Module v/s Total deformations with different pressure angle gears



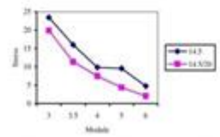
Graph 3 Comparison of equivalent stresses between two different asymmetric spur gears (14.5/20°) (20/30°) with different modules



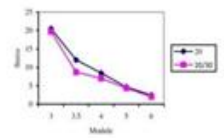
Graph 6 comparison of total deformation between symmetric (20°) and asymmetric (20/30°) spur gears with different modules



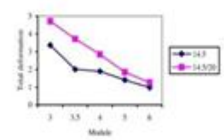
Graph 4 comparisons of total deformation between two different asymmetric spur gears (14.5/20) (20/30) with different modules



Graph 7 comparisons of equivalent stresses between symmetric (14.5°) and asymmetric (14.5/20°) spur gears with different modules



Graph 5 comparisons of equivalent stresses between symmetric (20°) and asymmetric (20/30°) spur gears with different modules



Graph 8 comparisons of total deformation between symmetric (14.5°) and asymmetric (14.5/20°) spur gears with different modules

VI . CONCLUSION

Spur gears that are symmetric have been created and validated. The Hertz theory of contact stresses is used to estimate contact stresses. With the aid of ANSYS and CATIA, symmetric and asymmetric spur gears have been modelled and analysed. The following conclusions have been drawn from the data:

1. The tension placed on the gear teeth is decreased by increasing the module for the same gear ratio.
2. The strength of the teeth has enhanced as a result of the module tooth's proportionate rise in size.
3. The tension exerted in the gear teeth decreases with increasing pressure angle.
4. The thickness at the tooth root increases as the pressure angle increases. With time, the strength also becomes better.
5. For both high module and low pressure angled gears, the overall deformation is minimal.
6. The graph finally shows that asymmetric spur gears are stronger than symmetric spur gears.
7. A 20/30° pressure angled spur gear is more robust than a 14.5/20° pressure angled gear when it comes to asymmetric spur gears.
8. Compared to gear ratio 1, the overall deformation and strains generated in the spur gears with ratio 2 are larger.
9. Consequently, we also draw the conclusion that asymmetric spur gears are less stressed.

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